The physics and mathematics of javelin throwing

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Abstract

Javelin flight is strictly governed by the laws of aerodynamics but there remain well-entrenched but misleading views about the factors involved. This paper gives a simplified but accurate view of the physics and dynamics of javelin flight and describes a software implementation of these.

1 Overview

This paper was written after realising that there were still many misunderstandings about the actual physics of throwing in general and javelin throwing in particular. The author is nowadays a computer scientist and UKA level 2 javelin coach but was a competent thrower when younger, so this may help both coaches and athletes to understand what is going on. The physics will be described first followed by a mathematical model and finally by a description of a freely available modelling software package which embodies these principles.

The software will only model post-1986 javelins. The essential difference was that the centre of gravity was moved forward by 10cm in 1986 because of the prodigious distances being achieved by such throwers as Uwe Hohn, ($>$ 104m.). The only other alternative would have been to move the javelin outside the main arena which would have deprived onlookers of seeing one of the most spectacular of all field events. The effect of this re-balancing is essentially two-fold:-

- The average distance is reduced by around 10%
- Moving the centre of gravity forward means that it is now about 6cm in front of the *centre of pressure*. The centre of pressure is defined to be the point at which the aerodynamic forces of lift and drag on the javelin apply. This means that there is an upward lifting force 6cm aft of the downward force acting through the centre of gravity which is situated around the front edge of the handle. The effect is that the javelin experiences a turning moment in the vertical plane which forces the point down and which therefore causes it to stick in removing any ambiguity as to where

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it has first landed. The women's 600gm javelin was similarly adjusted in 1999. Unfortunately, the disastrous current performance of many 700 gm javelins is because no such steps have been taken with this javelin with the result that the manufacturers seem to ignore it presumably to promote greater distances. Unfortunately, the flight characteristics would suggest that the centre of gravity and centre of pressure are effectively co-located meaning that it will very frequently land flat and the author has personally witnessed flat landings at 60m with most modern manufacturers in the English Schools with this most frustrating implement as well as half of all throws in the British Master's championships being flat landers, (M50 and M55 use the 700gm.)

Note that this is something of a simplification because the centre of pressure moves during the flight as the javelin changes its angle to the air-flow over it.

2 Some basic principles

The javelin is not so much a throw as a long pull. This is a most important lesson to impart to young athletes as they gradually begin to pick the event up. The *only* thing that matters is the speed of the javelin as it leaves the hand, assuming it is aligned properly. Furthermore, the distance it travels is proportional to the square of the speed so that an increase of 10% in the speed increases the distance by a factor of 1.1 x 1.1 = 1.21 or 21%. Of course increasing speed by 10% is harder than it sounds and is dependent on the biomechanical properties of the thrower.

So how is the javelin accelerated up to its final speed before leaving the hand ? In essence, the hand pulls on the javelin for a distance of some 2m in adult champion throwers accelerating the javelin from around 6 metres per second relative to the ground (allowing for the run-up) to around 30 metres per second relative to the ground. Note that Newton's law determines how successful this is as it is not simply the force which is applied. The musculature has to accelerate not only the javelin but also the athlete's arm and upper body. Newton's law states that the acceleration achieved is the force the athlete can apply divided by the mass which is being accelerated. If the athlete has a heavy musculature, the force which can be applied has to overcome the inertia of this mass also to achieve the maximum acceleration. This is why good javelin throwers are usually long and rangy. With respect to the muscles used in the upper arm, the bicep is simply added mass which hardly contributes as the bulk of the acceleration in this phase of the throw is applied by the tricep, so big biceps equal wasted mass.

3 The dynamics of the throw

Figure 1 shows the nomenclature used.

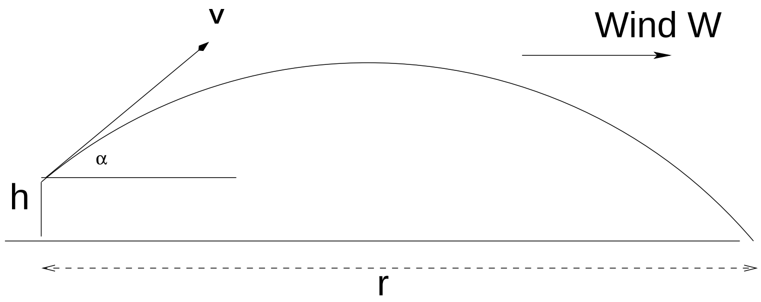

Figure 1: The nomenclature used in the accompanying text. A javelin is launched with velocity V at angle α from a height h m. above the ground with a wind of W m/s and flies a distance of r m.

3.1 Simple mathematical treatment

Let t_u be the time taken to reach the highest point of the flight from leaving the hand. Let t_d be the time taken to reach the ground from the highest point of the flight. Let s_u be the height reached above the point of release and let g be the acceleration due to gravity. Let h be the height at which the javelin is released. Let r be the distance covered by the javelin.

Then, resolving upward, the time taken to reach maximum height resisted by gravity is given by

$$0 = V sin\alpha - g t_u \tag{1}$$

Knowing V, α and g then gives t_u. The maximum height reached above the hand is given by:-

$$0 = V^2 sin^2\alpha - 2 g s_u \tag{2}$$

The time taken to reach the ground from maximum height is given by

$$s_u + h = 0 + \frac{1}{2} g t_d^2 \tag{3}$$

Knowing h, s_u and g then gives t_d. The total time the javelin is in the air is therefore $t_u + t_d$. The horizontal distance covered by the javelin is therefore given by:-

$$r = V sin\alpha (t_d + t_u) \tag{4}$$

This gives the classic parabolic shape of an ideal body in flight and is a useful approximation to the flight of a javelin. To do it more accurately however and account properly for wind and all the other factors, the aerodynamics must be re-introduced.

3.2 Enhanced mathematical treatment including aerodynamics

Drag, lift and pitch Linearised theory for airflow around an object like a javelin ([7]) allows us to approximate the drag force when the javelin is parallel to the wind as

$$D = 2\gamma\pi\rho V^2 \epsilon^2 \tag{5}$$

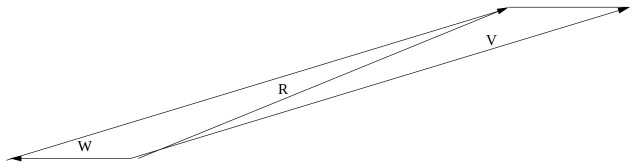

Figure 2: The parallelogram of velocities when a javelin is launched with velocity V at angle α to the ground into a wind of W m/s. The resultant velocity is R

where ϵ is the radius of the head of the javelin, ρ is the density of air and γ is a constant. There is no lift in this case. Pitch (or attack) where the javelin makes a small angle with the wind can be simulated by increasing this value to represent more of the javelin's aspect being presented to the air-flow around it. A simple calculation reveals that the increased area presented assuming the javelin is cylindrical with radius ϵ (not a bad assumption given that its tapered at both ends and thicker in the middle) is given by

$$\pi\epsilon^2 + 4L\epsilon sin(\delta) \tag{6}$$

where L is the length of the javelin and δ is the angle of attack. The total longitudinal drag is therefore given by

$$F_D \approx 2\gamma\rho V^2(\pi\epsilon^2 + 4L\epsilon sin(\delta)) \tag{7}$$

Resolving vertically, the angle of attack also gives a perpendicular lift

$$F_L \approx 2\gamma\rho V^2(4L\epsilon sin(\delta))sin(\alpha) \tag{8}$$

These terms need to be incorporated into the governing equations of motion.

More on attack Attack is a little more subtle than it appears. First of all, it is not obvious that a zero angle of attack produces the longest distance because the increased drag can be offset by increased aerodynamic lift although if the attack angle is too large, the javelin stalls and just falls out of the air. Second, the effects of relative motion must be taken into account. In its simplest form, we think of launching the javelin by throwing through the point and we might be tempted to think that the attack angle is then zero and there is minimum lift and drag. However, this neglects the parallelogram of velocities as shown in Figure 2. The effect of the headwind shown is to reduce the velocity of the javelin but also *to change its direction* so that there is a non-zero attack angle. The effect is exactly the same as launching the javelin with a positive attack angle into a zero wind. If zero relative angle of attack leads to the optimum distance, then in the case of a headwind, this means launching the javelin with a slight down-angle of attack so that its resultant angle with the wind is zero.

Analysis of a number of champion throwers often shows the javelin launched with a slight down-angle of a few degrees. The brief discussion above suggests that this will be beneficial only in a head-wind. The resultant velocity R and the effective angle of attack can be found using the cosine and sine rules.

Javelin types To complicate things further, javelins come in basically three types, Headwind, Tailwind and Neither (General). Javelin manufacturers are not allowed to move the centre of gravity and it is checked at weigh-in. The only significant remaining parameter they have some control over is then the centre of pressure. In the mid-80s one attempt to influence this was made by producing roughened tails which move the centre of pressure in such a way as to simulate javelins similar to the pre-1986 types. Since this undermined the whole purpose of the 1986 re-modelling, it was quickly banned. Nowadays only modest changes can be made which are related to the nature of the prevailing wind. To summarise, the three kinds of javelin have the following properties:-

- Headwind. Pointed javelins which it is commonly believed fly a little further into a headwind but will not fly as well as a tailwind javelin into a tailwind.
- Tailwind. Blunter nosed javelins to attempt to break the boundary layer flow around the javelin a little earlier with the effect of bringing the centre of pressure a little closer to the centre of mass giving a smaller pitching moment to keep the nose up a little longer. The common belief is that these function better in a tail wind.
- General. These javelins are usually pointed and are not optimised in any particular way.

In the author's experience, folklore about the behaviour of these javelins abounds but the mathematical model which follows does not support the folklore which the author found puzzling for some time. Certainly headwind and general javelins have less drag because the value of ϵ is smaller but the centre of pressure also moves so there is a counterbalancing effect. This was finally cleared up by [5] in a personal communication describing recent comments about Headwind and Tailwind javelins made by one of the pioneers of modern javelin design, Dick Held. In essence, Held made it clear that the javelins were originally distinguished only by a blunter nose - the shafts were identical. Apparently nobody would throw the blunt javelin because it would obviously increase the drag, however *Held knew that the blunt javelin outperformed the sharp javelin in almost every environment*, (for very similar reasons to the higher performance of the rough-tailed javelin but not as emphatic). In order to get people to use it, he called it a Tailwind and people then began to use it in tail winds and so the belief grew.

Full equations of motion This section simply turns the above physical factors into mathematical form so that they can be used to predict the flight of a javelin. Let m be the mass of the javelin, δ be the attack angle above the angle of delivery and let the position vector of the javelin relative to the delivery line be (r,s,q) where r is in the direction of the javelin flight, s is upwards and q is across from right to left. Resolving in each of these three directions using Newton's law (Force = mass x acceleration) and including the lift and drag terms leads to the following coupled non-linear differential equations:-

$$m\frac{d^2s}{dt^2} = -mg + \beta_j sin(\delta)sin(\alpha)((\frac{dr}{dt})^2 + (\frac{ds}{dt})^2) \qquad (9)$$

$$m\frac{d^2r}{dt^2} = -(\gamma_j + \beta_j ABS(sin(\delta)))((\frac{dr}{dt})^2 + (\frac{ds}{dt})^2) \tag{10}$$

$$m\frac{d^2q}{dt^2} = 0 \tag{11}$$

where

$$\beta_j = 8\gamma\rho\epsilon L \tag{12}$$

and where

$$\gamma_j = 2\gamma\pi\rho\epsilon^2 \tag{13}$$

Here β_j and γ_j are fitting constants *which depend on the javelin type essentially through the point radius* ϵ. A simple magnitude analysis gives $\gamma_j \simeq \frac{\epsilon}{3}\beta_j$. These equations are integrated forward in time using a simple Kutta-Merson procedure with control over global error, see for example, [6] to give the position vector (r,s,q) at all times after launch. The initial conditions at t = 0 are:-

$$r = 0 \tag{14}$$

$$\frac{dr}{dt} = V cos(\alpha) \tag{15}$$

$$s = h \tag{16}$$

$$\frac{ds}{dt} = V sin(\alpha) \tag{17}$$

$$q = 0 \tag{18}$$

$$\frac{dq}{dt} = W_x \tag{19}$$

where W_x is the crosswind.

Pitching moment Finally the pitching moment is included by basically solving:-

$$r\frac{d\alpha}{dt} = -moment \tag{20}$$

concurrently with the equations above, where the moment is the moment of the lifting and drag forces about the centre of gravity. The movement of the centre of pressure has to be parametrised as it is related to the generally unknown behaviour of the boundary layer around the javelin during flight.

An example of two trajectories for a Tailwind javelin thrown into a brisk headwind is shown in 3. The lower trajectory is delivered with no attack and the higher trajectory is delivered with a 5 degree attack angle. The stalling effect can be clearly seen. The qualitative features of javelin flight seem well described by the model and predictions match the observations of [4] well where relevant data was given.

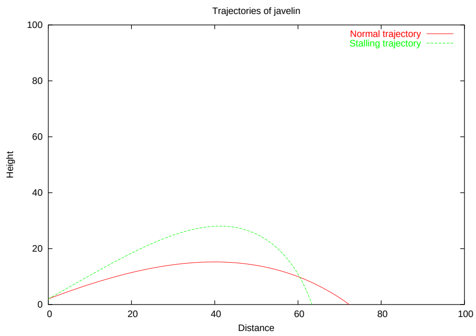

Figure 3: Illustrates the effects of drag on a high trajectory with attack into a head wind.

Rotation One factor which was omitted because it appears to be small is the axial rotation of the javelin. For a right hander, coupled with the pitching moment, the javelin will be deflected slightly to the left. It is relatively easy to incorporate this and will be included at a future stage. The axial rotation has been reported as high as 25 revolutions per second. The reason for the deflection is the gyroscopic principle of precession based on the conservation of angular momentum. There are two rotations, one in the vertical plane when the javelin point pitches downwards and the axial rotation. For a right handed thrower, the javelin precesses to the left if the axial rotation is clockwise from the back. It is important to note however that high axial rotation resists the downward pitching moment and will intoduce a slight drag and sideways lift effect. Whether this increases the horizontal distance travelled or not is currently unknown. It may be possible to exploit this by using different grips but this was not considered further here. It is likely however that the horseshoe grip will lead to slower axial rotation than either of the other two conventional grips.

Stiffness One other factor which was not incorporated is the effect of stiffness of the javelin. If a flexible javelin is mis-hit, a significant amount of the power is expended in oscillating the javelin, which ultimately dissipates as heat warming the javelin up slightly. The author can recall an extreme example of this from the current world record holder, Jan Zelezny where in one series, the javelin was mis-hit so badly, it appeared as though it would break up in mid-air such was the amplitude of the oscillation. The javelin only travelled about 40m. The next throw from Zelezny was around 90m. For a stiff javelin, there will be a lesser effect. If the javelin is not mis-hit, it doesn't matter. Since this only affects poorer throws, it will not be considered further here.

4 Effects of biomechanical factors

A considerable amount of work has been done on this using for example, the idea of kinetic chains by Bartlett, Morriss and others, [1], [3], [4]. In this analysis, we will consider a very simplified view and will only be interested in a straight trade-off between the angle at which the javelin can be launched and the distance the thrower can hang on to it during the acceleration phase. The simplest

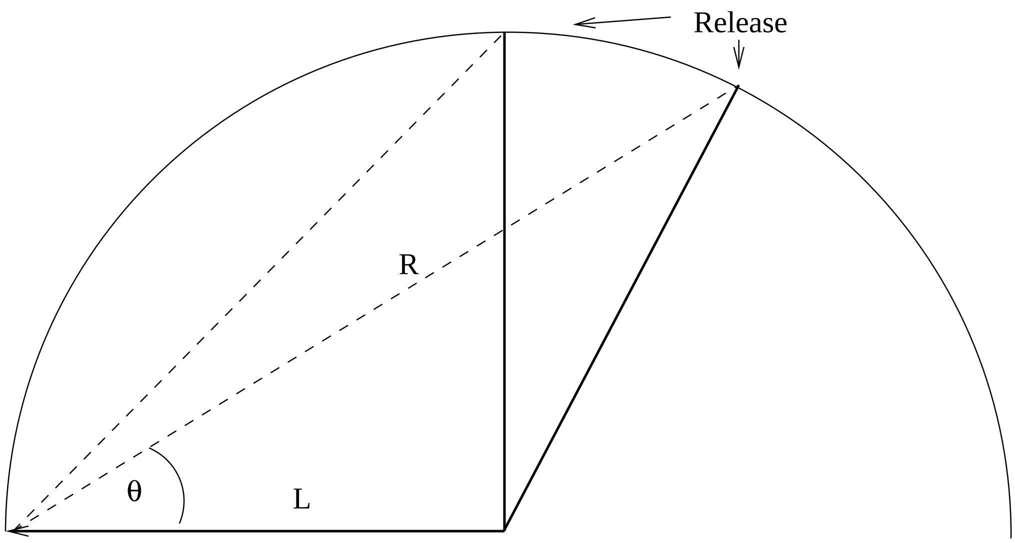

Figure 4: Illustrating the trade-off between increased angle of delivery θ and length of pull R which the javelin thrower can apply. Note that the movement forward of the shoulder pivoting on the base of the back and trunk can be modelled simply by increasing the 'length' of the arm, L.

representation of this is shown in Figure 4. In this diagram, using the cosine rule,

$$R^2 = 2L^2 - 2L^2 cos(180 - 2\theta) = 2L^2(1 + cos(2\theta)) \tag{21}$$

This can be simplified to give

$$R = 2Lcos(\theta) \tag{22}$$

In essence, this states as the delivery angle increases, the range along which the javelin can be pulled gradually diminishes. This is used in the software package itself to optimise the delivery parameters.

5 Conclusion

The above set of equations handle the qualitative behaviour of a javelin well and are included in the software package *Javelin Flight Analyser*, [2]. A screenshot of the front page is shown as Figure 5. The package is freely available as a self-installing executable for Windows 98/2000/XP from the quoted web location.

6 Acknowledgements

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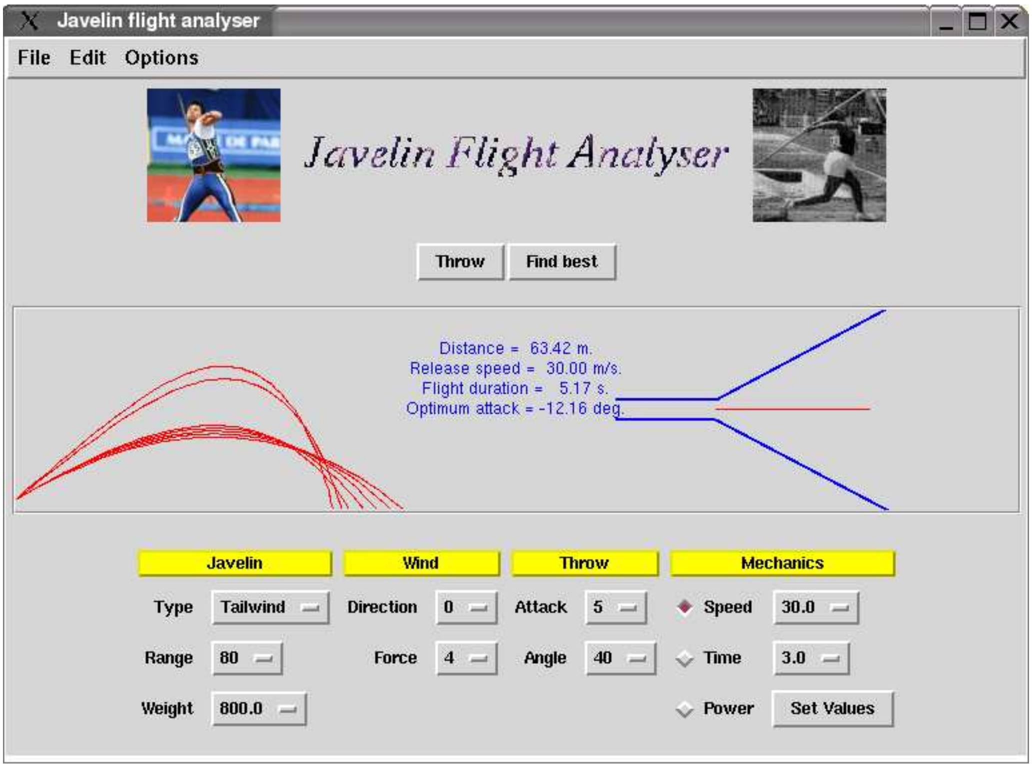

Figure 5: A screenshot of the front page of the Javelin Flight Analyser. The effects of five angles of attack at a launch angle of 30 degrees and two at 40 degrees are shown, illustrating the non-parabolic effects of drag.

References

[1] R.J. Best, R.M. Bartlett, and C.J. Morriss. A three-dimensional analysis of javelin throwing techniques. *Journal of Sports Science*, 11(4):315–328, August 1993.

[2] L. Hatton. Javelin flight analyser. *http://www.leshatton.org/javelin_2005.html*, May 2005.

[3] C. Morriss and R. Bartlett. Biomechanical factors critical for performance in the men's javelin throw. *Sports Medicine*, 21(6):438–446, June 1996.

[4] C.J. Morriss, R.M. Bartlett, and N. Fowler. Biomechanical analysis of the men's javelin throw at the 1995 world championships in athletics. *Thrower*, 78:18–31, July 1998.

[5] B. Parkes. Notes on head and tailwind javelins by Dick Held dated 7 june, 2004. *Personal Communication*, May 2005.

[6] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery. *Numerical Recipes in C*. Cambridge University Press, 2nd edition, 1999. ISBN 0-521-43108-5.

[7] Milton van Dyke. *Perturbation methods in fluid mechanics*. Academic Press, 1970.