Computing the distance of a hammer throw

Les Hatton
Oakwood Computing Associates Ltd
www.leshatton.org, lesh@oakcomp.co.uk

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Abstract

A simple but reasonably accurate formula is derived for computing the distance travelled in a hammer throw.

1 Background

Computing the flight of a javelin is a complex process requiring the use of sophisticated aerodynamic models, [3], [4], [1], [5], [2]. The javelin is strongly affected by wind, angle of attack, delivery angle, athlete biomechanics, javelin vibration, javelin type, and a number of other factors. The discus is similarly sensitive to aerodynamical influences and predicting its flight is also a complex process.

In contrast, the hammer is very much less affected and is close to the standard ballistic solution for a thrown weight in a vacuum. The only parameters necessary for a fairly accurate prediction of the distance travelled are the delivery angle relative to the ground (α), delivery speed (V) and the height above ground (h) when the hammer is released.

To compute this, we calculate the total time the hammer is in the air using the vertical equations of motion and then use this total time in the horizontal equation of motion to get the distance.

Time to reach greatest height Let T be the time to reach the maximum height. The hammer is released with a vertical velocity component $V.sin(\alpha)$. The time taken to reach its maximum height is given by using the equation v = u + gT, where v is the final velocity, u the initial velocity and g the acceleration due to gravity, (= -9.8 ms^{-2} in the positive y direction).

$$0 = V.sin(\alpha) + gT \tag{1}$$

giving

$$T = -\frac{V.sin(\alpha)}{g} \tag{2}$$

The hammer will reach a height s given by $v^2 = u^2 + 2gs$. This is therefore:-

$$s = -\frac{V^2 sin^2(\alpha)}{2g} + h \tag{3}$$

, where I have added h, the height at which it starts. The time T' taken to fall from this height s back down to the ground is given by $s = uT' - (1/2)gT'^2$. Which gives finally

$$-\frac{V^2 sin^2(\alpha)}{2g} + h = 0 - (1/2)gT'^2 \tag{4}$$

which simplifies to:-

$$T' = \sqrt{\frac{V^2 sin^2(\alpha)}{g^2} - \frac{2h}{g}} \tag{5}$$

The total distance D travelled by the hammer is therefore given by the horizontal equation of motion:-

$$D = V.cos(\alpha)(T + T') = V.cos(\alpha)[-\frac{V.sin(\alpha)}{g} + \sqrt{\frac{V^2 sin^2(\alpha)}{g^2} - \frac{2h}{g}}] \tag{6}$$

Finally writing g = -9.8 ms^{-2}, and re-organising a bit, we get:-

$$D = \frac{V^2 sin(2\alpha)}{9.8}[\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{9.8h}{V^2 sin^2(\alpha)}}] \tag{7}$$

In this form it can be easily seen that this reduces to the ballistic solution as $h \rightarrow 0$ as it should. Also, the higher up you are, the further it goes horizontally as would be expected. The correction term in the square root is perhaps 1-2% for a typical 75m throw. Since this adds to the distance whereas aerodynamic drag reduces distance, these will balance to a certain extent in practice improving the quality of the ballistic solution.

Optimal throw Note that releasing it off the ground means that the optimum delivery angle is slightly less than 45 degrees even in a vacuum.

References

[1] R.J. Best, R.M. Bartlett, and R.A. Sawyer. Optimal javelin release. *Journal of Applied Biomechanics*, 11:371–394, 1995.

[2] L. Hatton. Javelin flight analyser. *http://www.leshatton.org/javelin_2005.html*, May 2005.

[3] M. Hubbard and L.W. Alaways. Optimum release conditions for the new rules javelin. *International Journal of Sport Biomechanics*, 3:207–221, 1987.

[4] M. Hubbard and S. Laporte. Damping of javelin vibrations in flight. *Journal of Applied Biomechanics*, 13:269–286, 1997.

[5] C. Morriss and R. Bartlett. Biomechanical factors critical for performance in the men's javelin throw. *Sports Medicine*, 21(6):438–446, June 1996.